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Methods for improving vehicle interior noise FEM-PEM simulations in the mid-frequency range

Doctoral theses

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2023

1. Research Summary

1.1 Motivation

Continuous development in the automotive industry has led customers to expect quieter and more efficient vehicles for their daily commute. Megatrends, such as the need to reduce the environmental footprint of vehicles, urbanisation or the general increase in the need for mobility have shifted paradigms for vehicle producers. Buyers today prefer intelligent, efficient and comfortable cars instead of vehicles requiring the increased level of driver control or exposure to tiring noises, vibrations as well as harsh, intrusive effects in the cabin. As a result of the aforementioned effects, today's mainstream vehicles require engineering sophistication that was formerly typical for luxury cars only. This manifests itself in the need for accurate engineering methods to achieve the desired vehicle characteristics, rather than just the usage of fine materials. To satisfy customers, highly scientific methods have to be applied during the design process of cars. Today, the noise, vibration and harshness (NVH) level is one of the key vehicle properties customers care of. However, the sole development goal is not, in all cases, the absolute reduction of noise. While mainstream vehicle models are expected to be quieter and smoother, their performance-oriented versions need to have a unique sound character to them, to provide an aural excitement beside driving enjoyment. Both of these development targets mean that achieving the desired acoustic characteristics has gained huge importance within the vehicle development process. Complex simulations and verification measurements have to be carried out to ensure that the final product meets the development requirements and that the development process itself is ready to flexibly engineer outstanding products.

In the current transformation of the automotive landscape, the significance of acoustic methods will not decline. Simply put, just changing the internal combustion powertrain to an electric one will not solve the many acoustic challenges that arise from the vibrations and noises from the various mechanical systems and load exchange flow-induced noises of the conventional power unit. If anything, the significance of accurate and efficient acoustic prediction methods in industry is going to grow. Electric drivetrains introduce fundamentally new challenges with their distinctive acoustic behaviour. Planetary gearsets inside electric motors emit noise at much higher frequencies than internal combustion counterparts, while the rest of the high-voltage system can also be a source of unwanted disturbances. Not to mention that the absence of the base noise emission of an IC engine brings into spotlight previously difficult to notice operating noises of the HVAC system, various servo motors serving the comfort of passengers or the tire- and wind noises at lower speeds. This means that acoustic treatments are expected to perform well not only at low frequencies (where chassis noises "masked" earlier by the combustion powertrain will now be audible) but also at high frequencies, where the excitation from e-motors will dominate (Lennström et.al., 2013). Alongside the improved acoustic performance, however, stricter efficiency goals will dictate continuous weight savings as well. This means that acoustic packages must also be designed in a greatly targeted fashion, applying only the necessary amount of material to highly specified locations. Both above mentioned, conflicting requirements coupled with the accelerated development cycles necessitate accurate numerical methods to determine acoustic properties in the early phases of the design process.

Computer aided design and development has become industry standard, and especially in the case of extensive sound package design, numerical simulations are essential for timely and cost-effective development. Depending on the investigated frequency range, various acoustic methods are available: for higher frequency applications, energy-based methods, such as Statistical Energy Analysis (SEA) are well-suited, while Finite Element Methods (FEM) are appropriate for the lower frequency ranges. Because of the fundamentally different approaches of these solvers and the large variety in the physical attributes of acoustic systems, there is no set frequency boundary for their reliable operation. Therefore, in the so-called mid-frequency range (between 400-1000 Hz), both suffer from limitation. In this range, structures generally do not possess a high enough modal density for SEA and other energy-based methods to be applicable, while the FEM's frequency range is limited from above by statistical and modal effects. This means, that the high modal density increases the difficulty to identify individual modal contributions in the case of modal frequency response calculations, while the statistical uncertainties in the model also have a more pronounced effect on higher frequency behaviour. Unfortunately, from an excitation standpoint, this frequency range cannot simply be ignored, as disturbing sources exist that emit in these specific regions and the human ear's acoustic reception is also particularly sensitive in the mid-frequency gap.

Sound packages have become complex since their deliberate introduction in automobiles (Figure 1.), and with their improvement the underlying design practices and engineering methods have followed suit. FEM provides an ideal framework for the abstraction of these packages as it gives almost complete freedom for the available geometries. The main material constituents of modern automotive sound packages are foam- and felt-like materials, which are collectively known as Poroelastic Materials (PEM), since they consist of a solid, porous matrix filled with air. After modelling just their mass with non-structural mass elements (the so-called Non-Structural Mass method - NSM), a significant breakthrough was the application of the Biot-theory to model absorptive effects as well. With a variety of parameters, the Biot-parameters based trim modelling allow a high degree of fine-tuning of the acoustic packages of vehicles. However, the determination of these parameters, as well as their behaviour over the various frequency ranges poses several challenges on its own. As such, the combination of finite element method and poroelastic materials into FEM-PEM modelling still opened up significant ways in improving interior acoustic simulations. However, even the addition of the Biot model did not solve the mid- and high-frequency discrepancies between simulation and measurement results.



Figure 1.: Complex interior of a state-of-the-art luxury limousine with elaborate trims (Audi Media Center, 2023).

Despite the challenges of using FEM, it is worthwhile to attempt broadening this method's applicable frequency range. By its vast applicability to structural engineering problems, it has become a standard tool in product engineering pipelines, especially in the automotive industry. As model preparation is one of the major industrial challenges in numerical simulation methods, the use of an already established method, which duplicates use cases for a single model preparation step, shows huge economic potential.

Therefore, the general goal of this work is to investigate the applicability of FEM-PEM methods for predicting the mid-frequency interior acoustic phenomena in automotive structures. In particular, various phenomena will be investigated that limit the applicability of FEM for mid-frequency acoustic problems, such as the frequency dependency of Biot-materials, as well as the potential of an overdeveloped model that is too sensitive for manufacturing tolerances, with the addition of the design of test apparatuses that aid such a research methodology.

1.2 Modal frequency analysis in FEM

Modal frequency response analysis extends modal analysis by using mode shapes to uncouple the equations of motion for the structure and compile frequency response results. A trade-off compared to direct frequency analysis is that eigenmodes of both the structure and the fluid have to be calculated before the frequency response calculation. However, modal analysis is a mathematically less intensive method, and the calculated modal bases can be used for subsequent calculations, as long as the material and/or geometric properties of the structure and the fluid do not change. As the current investigation focused almost exclusively on varying the material parameters of trims, this advantage of modal frequency response was fully utilized. Mathematically, the following algorithm is used in the applied solvers. The well-known equation of motion to be solved in FEM is as follows:

$$(-\omega^{2} [M] + i\omega[B] + [K]) \{u\} = \{P(\omega)\}$$
(11)

It is evident here, that the direct solution would involve inverting a large matrix, hence its enormous computational load in the case of millions of degrees of freedom (which can easily occur in a full-scale vehicle). As eigenmodes of a structure are distinct for each eigenfrequency, this orthogonality can be exploited to transform the problem from physical coordinates to modal coordinates, using:

$$\{x\} = [\phi]\{\xi(\omega)\}e^{i\omega t} \tag{12}$$

Above equation only holds if all modes are computed, meaning that the equations are only fully uncoupled if all modes of the structure are included in the calculations. However, for a known excitation in a specific frequency range, a truncated modal base can provide sufficient information, since the contribution of modes far beyond the frequency range of interest is very small. For the structures of interest, a 180% of the upper frequency spectrum limit was used (this means that in the current case, modes were computed up to 1800 Hz for investigations running up

to 1000 Hz). To account for the truncation error, residual vectors are calculated. With damping temporarily ignored, substituting physical coordinates for modal ones and pre-multiplying with $[\phi]^T$ uncouples the equations of motion and yields:

$$(-\omega^{2} [\phi]^{T} [M] [\phi] \{\xi(\omega)\} + [\phi]^{T} [K] [\phi] \{\xi(\omega)\}) \{u\} = [\phi]^{T} \{P(\omega)\}$$
(13)

By transforming the equations of motion to modal coordinates, a set of singledegree-of-freedom equations have to be solved for modal responses, which are in turn summed to compute the physical responses in complex form (with magnitude and phase).

Presence of structural and viscous damping generally means that the equations of motion are coupled. To circumvent this problem, two possibilities are present. Either the full, coupled equations of motion for the damped system are expressed in modal coordinates and solved directly, or modal damping can be attributed to individual modes (NASTRAN, 2014). To increase computational accuracy and allow for a more detailed investigation, the first option was used for the structural computations conducted in this research campaign, while for the fluid cavities, modal damping was used. This latter simplification was justified by the homogeneity of the fluid, as it was in all cases air.

For modelling trim, the Biot model of PEM was employed, which describes porous media with 11 parameters in its formulation in VPS and NASTRAN-PEM. This equivalence of material model ensures better comparability between the solvers and eliminates conversion errors. Wave propagation within the phases is calculated using parameters of the solid and fluid phases, as well as coefficients that quantify the level of interaction between them. Automotive trim materials, such as foams, felts and fleeces all are bi-phasic materials and can be modelled in acoustic simulations using Biot-theory.

For the given cases in the modal frequency response analysis, simulations consisted of determining the modal bases for the structure and the fluid parts,

calculation of trim impedances and solving the frequency response function. Structural and fluid modal bases were re-used for multiple trim configuration calculations, while trim impedances could be re-used if the structure or the fluid was modified. Therefore, computations could be automated to a certain degree.

As different apparatuses were used for calculations, different meshes were applied. In all cases, plates and stiffeners were modelled using shell elements, while trims and the interior fluid cavity were represented as solids. For structures, 10 mm target element size was set for quad shell elements. The fluid cavity and trims were meshed with tetrahedral solid elements, with a 10 mm target size for the trims and 30 mm target for fluid medium. Material damping was applied to structural elements, while modal to the fluid cavity. Riveted and welded connections were modelled accordingly in the structural model of the metal cavity.

1.3 Biot-theory

As in the case of solving the Navier-Stokes equations in Computational Fluid Dynamics (CFD), the new method was not developed fully from scratch for this purpose, rather it was the application of a method used in another discipline to acoustic problems that brought a significant advancement in trim modelling. This method was the application of the Biot-theory in acoustic simulations. Biot-theory, or the theory of poroelasticity was originally developed by Maurice Biot in the 1940's for modelling wave propagation in porous materials composed of a solid matrix filled with a fluid (Biot, 1956). The theory found its initial use in determining settlement of structures on porous soils saturated with fluid in the pursuit and discovery of oil fields. Since the theory aims to describe the elastic behaviour of a porous medium, it is also named as the theory of poroelasticity, and thereby such two-phase materials applied in vehicles are also called Poroelastic Materials (PEM). Three underlying physical theories lay the fundaments of Biot theory: linear elasticity (of the solid matrix), Navier-Stokes equations (to describe the viscous fluid) and Darcy's law (to account for fluid flow in the pores). Altogether these theories are used to assemble the governing equations of the theory of poroelasticity, which describe wave propagation in porous media. Three waves are assumed to propagate through the material: a shear component within the solid matrix, as well as pressure components in the solid matrix and the fluid medium. The original formulation uses six translational variables $-u^s$, u^f vectors for solid and fluid displacements – to express strain- and kinetic energies as well as stress tensors.

The resulting set of three equations for the displacements, pressure and stresses, often referred to as the u-p formulation, are written below:

$$\nabla \sigma^{s} (\boldsymbol{u}^{s}, \boldsymbol{p}) = -\boldsymbol{\omega}^{2} \, \widetilde{\boldsymbol{\rho}}_{s} \boldsymbol{u}^{s} - \boldsymbol{\gamma} \nabla \boldsymbol{p} \tag{8}$$

$$\nabla \mathbf{p} = -\omega^2 \, \frac{\tilde{\rho}_{22}}{R} \mathbf{p} - \omega^2 \frac{\delta_{22}}{\theta^2} \gamma \boldsymbol{u}^s \tag{9}$$

$$\sigma^{s}(\boldsymbol{u}^{s},\boldsymbol{p}) = \delta\left(\boldsymbol{K}_{b} - \frac{2}{3}\boldsymbol{N}\right)\nabla\boldsymbol{u}^{s} + 2\boldsymbol{N}\boldsymbol{\varepsilon}^{s}(\boldsymbol{u}^{s})$$
(10)

, wherein

$$\boldsymbol{\gamma} = \boldsymbol{\theta} \left(\frac{\tilde{\rho}_{12}}{\tilde{\rho}_{22}} - \frac{\tilde{\boldsymbol{\varrho}}}{\tilde{\boldsymbol{\rho}}} \right), \, \tilde{\boldsymbol{\rho}}_s = \tilde{\boldsymbol{\rho}}_{11} \frac{\tilde{\rho}_{12}^2}{\tilde{\rho}_{22}} \tag{11}$$

Various formulations exist for determining the complex effective density as well as the compression modulus. Of these, however, the Johnson-Champoux-Allard formulation is most commonly used in commercial codes, such as those used in this work (Allard and Atalla, 2009). As a result, the following material parameters are required to describe poroelastic material behaviour in the discussed formulation (Figure 2. illustrates a generic cross-section):

- 1. Phase parameters of constituting materials:
- 2. Fluid density
- 3. Speed of sound in fluid
- 4. Solid matrix structural density: full mass divided by full volume
- 5. Specific, porous elastic material parameters:

- 6. Porosity: ratio of solid and fluid phase volume
- 7. Flow resistivity: describes viscous losses
- 8. Tortuosity: provides the coupling between matrix and fluid
- 9. Viscous characteristic length: another measure of viscous losses
- 10. Thermal characteristic length: describes thermal losses
- 11. Damping loss coefficient of the matrix: provides information on structural losses
- 12. Young's modulus of matrix: elastic behavior descriptor of the matrix
- 13. Poisson-coefficient of matrix: describes volume change under stress in the matrix



Figure 2.: Sample cross-section of a generic poroelastic material (Allard and Atalla, 2009).

2. Test Cases

Vehicle structures in general contain various joining methods, complex geometries, as well as a large number of modes. Therefore, on their own, vehicle bodies are not ideal candidates for methodology development. To thoroughly understand simulation methods, vehicle structures must be simplified into schematic but representative academic cases, where simulated components can be studied on lower complexity levels. However, the simplified geometries eventually must still include phenomena that are also present in real vehicle bodies. To conceptualize the multi-level complexity of the test apparatuses, it is important to identify the fundamental building blocks of vehicle bodies from an acoustical perspective. Three such blocks make up acoustic vehicle systems:

- Structure: consisting for the moment mainly of metal materials (steel or aluminium). Geometrically, structures can be divided into cosmetic (panellike) and load bearing (box section-like) elements.
- 2. Air Cavity: the enclosed air cavity interacts with the structure and the sound package, as well as provides the actual medium for sound transmission to occupants' ears.
- 3. **Trim:** all sound deadening porous-elastic components whose aim is to reduce interior noise levels are called trims, which are the main objectives of a FEM-PEM simulation.

A set of test cases have been identified, which follow each other in increasing complexity. These are illustrated in Fig. 3. below and explained in detail in the following sections. The logic was to test key concepts first on lower complexity models, followed by medium and full complexity models.



Figure 3.: Test cases 1-3 showing the increasing complexity of models.

A lightly damped structure is needed to see the most pronounced effects. A simple, homogeneous flat plate, Test Case 1, in a free-free condition is ideal for these investigations, with dimensions of 650 mm x 550 mm, and 2 mm nominal thickness.

Material is structural steel with nominal density of 7850 kg/m3, 210 GPa E-Modulus and Poisson's ratio of 0.3. Figure 4. shows the meshed FE-model of Test Case 1. This simple structure provides a perfect vessel to investigate purely structural response. While a flat plate as an academic case provides a great research vessel, it also highly magnifies the effects of trim materials. Without any reinforcement, a flat plate can be considered as a lightly damped structure, magnifying any possible changes that the use of either frequency-dependent trim material parameters or manufacturing tolerances would have on the acoustic response. These effects, however, may not be so pronounced for stiffer, heavier damped structures which more closely model a vehicle, therefore more representative test cases are required to draw conclusions.



Figure 4.: 650 x 550 mm sized flat steel plate with 2 mm nominal thickness served as Test Case 1, the most fundamental investigation platform.

An essential element of a real vehicle's interior acoustic behaviour is the interaction between the trim and the enclosed air cavity. To be able to study such phenomena, a whole vehicle could be considered a useful tool, however, with computing times in FEM on the order of days instead of hours for a large frequency domain, an intermediate solution was created. This intermediate solution took the form of a trapezoidal cavity, however, in two different configurations: one with acoustically rigid, non-participating walls, while another with acoustically flexible walls that are excited in the investigated frequency range and interact with the

interior air cavity. Both test cases, Test Case 2a with rigid and Test Case 2b with flexible walls had the same interior volume. To ensure rigidity for any frequency range, Test Case 2a was manufactured out of reinforced concrete, with a removable front concrete wall. Test Case 2b with acoustically participating walls was manufactured out of steel panels with varying thickness (Fig. 5. shows Test Case 2b in reality and with its FE-mesh). A detailed guide to the design and properties of these test cases will be shown later, in Section 3. Stiffener U-sections divide up the flat cavity boundaries, with the stiffeners riveted to the cavity walls. These ensure a similar acoustic behavior to a vehicle chassis. A good indicator for this is that the metal cavity has eigenmodes on the same order of magnitude as a real vehicle body-in-white. The sections on the side can be thought of as front and rear doors, while the other boundaries represent the firewall, windshield, rear end, floor and headliner sections of a real vehicle. 6 response points were selected inside the cavity to provide sound pressure level information.



Figure 5: Untrimmed cavity manufactured test specimen (left) and finite element model (right)

As the ultimate goal of the study was to draw conclusions for a full vehicle structure, the final test case, Test Case 3 was a fully trimmed vehicle structure, provided by Audi Hungaria, one of the key partners of the research effort. As a prominent contender in the premium segment, where interior acoustics is one of the key purchase decision factors for customers, the provided vehicle model employed significant amounts of porous materials for sound deadening purposes. The investigated vehicle was a 2015 Audi A3 Limousine in trimmed body configuration. This included a complete interior model with all trim elements, such as door cards, instrument panel, headliner, rear seats and carpeting. Test Case 3 was only investigated in fully trimmed configuration. Whole vehicle and its FE-representation are shown on Figure 6.



Figure 6.: Test Case 3 shown in reality (top) and as a FE-representation (bottom) (Buy-acar, 2023).

To understand the effect frequency dependent Biot parameters have, sound absorbing materials (trims) are applied to both apparatuses, as the third complexity level conceptual block. Applied trim was the same for test cases 1, 2a and 2b, composed of an absorbing foam with 55 kg/m³ density and thickness of 20 mm. Biotparameters of the foam are standard industrial values. Test Case 3 uses foams that were applied to the series production vehicle with manufacturer provided material parameter values for density, geometry and all relevant Biot parameters. While many of these parameters are frequency dependent, current investigation only considers the impact of frequency dependent Young's modulus and damping. Both the frequency dependent Young's modulus, as well as the frequency dependent damping have been experimentally measured on a foam sample using a Dynamic Mechanical Analyzer (DMA). As the sample could only be excited in the 20-60 Hz region, time-temperature superposition was used to obtain the full material parameter curve for the entire frequency range. This parameter curve was then scaled for the various foam types in Test Case 3 as well to introduce frequency dependency in the full scale vehicle model.

For Test Case 1, the entire plate was covered with the applied trim, in three thicknesses: 10, 30 and 50 mm. Test Case 2, being a more complex geometry, replicated a real vehicle's fully trimmed stage. This means that trim was applied to the representative firewall, floor, front- and rear door card, trunk and headliner areas. In all cases, the same 20 mm foam trim was used. Figure 7. illustrates the trimmed models.



Figure 7: Trimmed plate and cavity models.

3. Thesis 1.

I have designed a scaled vehicle interior model for the purposes of acoustic FEM-PEM and SEA simulation validation in the mid-frequency (400 -1000 Hz) range. Three key design guidelines have been established to achieve this:

- a) The optimum volume of the cavity can be achieved when the criteria of minimum 3 fluid eigenmodes in third-octave frequency bands above 282 Hz center frequency (enabling SEA validation) is met, while minimizing the total number of fluid eigenmodes (enabling FEM-PEM validation).
- b) For validating fluid-PEM interaction, rigid walls are required. For a reinforced concrete structure, the wall thickness should be selected to find a compromise between the total number of structural eigenmodes and the minimum thickness dictated by the manufacturability.
- c) For validating fluid-structure-PEM interaction, acoustically flexible walls are required.

Relevant publications: [C]

4. Thesis 2.

Frequency-dependent damping and/or Young's modulus of poro-elastic materials (PEM) can significantly affect the amplitude and resonance frequencies of the response of geometrically simplified pure foam trim-structures. This effect was observed to be in the order of 3-5 dB difference in amplitude and 10-40 Hz shift in frequency up to 1000 Hz for the academic test cases considered in this thesis. However, for a complex vibro-acoustic system, such as a trimmed body passenger vehicle structure consisting of multi-material trim elements including foam, heavy layer and felt elements, it has been proven that their combined effect on the response is below the audible threshold of sound pressure level difference [2 dB(A)].

Relevant publications: [A, B, G]

5. Thesis 3.

In a mass production environment, the thicknesses of sheet metal components exhibit a statistical deviation from the nominal thickness values. It has been found, that this may have the following effects on the acoustic response in simulations:

- a) For structures with clearly separable load carrying elements (beams, stiffeners, frame, etc.) and non-load carrying panels made of sheetmetal, the deviations from the nominal thickness of the non-load caryying panels may significantly influence the SPL response in the interior volume enclosed by the structure. This effect was observable for the full frequency range up to 1000 Hz.
- b) For vehicle-like structures, where the load carrying elements (beams, stiffeners, frame, etc.) and non-load carrying panels made of sheetmetal are not clearly separable due tot he monocoque nature of the structure. the deviations from the nominal thickness had a significant impact on the interior SPL frequency response only above 400 Hz. This may contribute to the uncertainty of mid-frequency range FEM-PEM simulations.

Relevant publications: [B]

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